

# 数学试题参考答案

一、选择题 (每题 4 分, 共 40 分)

1-5 C C D C A    6-10 B D B B A

二、填空 (每题 4 分, 共 20 分)

11.  $\leq 3$  的数即可    12.  $(-3, 3)$     13.  $-2$     14.  $130^\circ$     15.  $37$

16. (1) 解:  $|1 - \sqrt{2}| - 2\sin 45^\circ + \left(\frac{1}{3}\right)^{-2}$   
 $= \sqrt{2} - 1 - 2 \times \frac{\sqrt{2}}{2} + 9 \dots\dots\dots 3$  分

$= \sqrt{2} - 1 - \sqrt{2} + 9$

$= 8 \dots\dots\dots 5$  分

(2) 解: 原式  $= \left(\frac{a+1}{a+1} - \frac{1}{a+1}\right) \div \frac{a^2 - 2a + 1}{a^2 - 1}$

$= \frac{a}{a+1} \div \frac{(a-1)^2}{(a-1)(a+1)}$

$= \frac{a}{a+1} \times \frac{(a-1)(a+1)}{(a-1)^2}$

$= \frac{a}{a-1}, \dots\dots\dots 8$  分

当  $a = -5$  时, 原式  $= \frac{-5}{-5-1} = \frac{5}{6} \dots\dots\dots 10$  分

17. (1) 证明: 由作图知  $MN$  垂直平分  $AB, AP = AG$

$\therefore BG = AG, BP = AP$

$\therefore AP = BP = AG = BG$

因此, 四边形  $APBG$  是菱形  $\dots\dots\dots 5$  分

(2) ∵ 四边形  $APBG$  是菱形

$$\therefore AQ \perp PG, AQ = \frac{1}{2} AB, GQ = \frac{1}{2} PG = \frac{1}{2} \times 10 = 5$$

在  $Rt\triangle AQG$  中,  $AQ^2 + QG^2 = AG^2, AQ^2 = 13^2 - 5^2 = 144$

$$\therefore AQ = 12$$

$$\therefore AB = 2AQ = 2 \times 12 = 24 \dots\dots\dots 10 \text{ 分}$$

18. 解: (1) 设每个圆形小灯笼直径为  $x$  cm, 两个圆形小灯笼圆心间距为  $y$  cm,

由题意得: 
$$\begin{cases} x + (10 - 1)y = 118 \\ x + (15 - 1)y = 178 \end{cases} \dots\dots\dots 4 \text{ 分}$$

$$\text{解得: } \begin{cases} x = 10 \\ y = 12 \end{cases}$$

因此, 每个圆形小灯笼直径为 10cm, 相邻两个圆形小灯笼圆心之间的距离为

$$12\text{cm}. \dots\dots\dots 6 \text{ 分}$$

$$(2) \text{ 函数关系式为: } L = 12n - 2 \dots\dots\dots 10 \text{ 分}$$

19. (1)  $m$  的值为 99,  $n$  的值为 94;  $\dots\dots\dots 4 \text{ 分}$

$$(2) \text{ 解: } 600 \times \frac{10}{20} + 400 \times \frac{6+8}{20} = 580 \text{ (人)}$$

则参加此次竞赛活动学生获得优秀(90 分以上)成绩的总人数为 580 人.  $\dots\dots\dots 8 \text{ 分}$

(3) 解: 八年级学生对“中文的历史发展”知识了解的更多.

理由: 八年级所抽学生的平均成绩大于七年级的平均成绩.(从中位数或者从众数的角度分析均可.)  $\dots\dots\dots 12 \text{ 分}$

20. 解: 如图, 过点  $E, F$  分别作  $EM \perp CD, FN \perp CD$ , 垂足分别为  $M, N$ , 过点  $E$  作  $EP \perp FN$ , 垂足为  $P$ .  $\dots\dots\dots 2 \text{ 分}$

$$\therefore \angle EPN = \angle PNM = \angle EMN = 90^\circ,$$

$\therefore$  四边形  $EMNP$  为矩形,

$\therefore EM=PN, \angle MEP=90^\circ .$

$\therefore \sin \angle ECM = \frac{EM}{CE} = \frac{EM}{120} = \sin 75^\circ \approx 0.97, \dots\dots\dots 4 \text{分}$

$\therefore EM=116.4 \text{ (cm)},$

$\therefore PN=116.4\text{cm}. \dots\dots\dots 6 \text{分}$

$\because \angle EMC=90^\circ , \angle ECM=75^\circ ,$

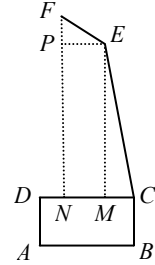
$\therefore \angle CEM=15^\circ ,$

$\therefore \angle FEP = \angle CEF - \angle PEM - \angle CEM = 135^\circ - 90^\circ - 15^\circ = 30^\circ ,$

$\therefore PF = \frac{1}{2}EF = \frac{1}{2} \times 40 = 20(\text{cm}), \dots\dots\dots 8 \text{分}$

$\therefore PF+PN+BC=20+116.4+25=161.4 \approx 161 \text{ (cm)},$

$\therefore \text{点 } F \text{ 到地面的高度约为 } 161\text{cm}. \dots\dots\dots 10 \text{分}$



21. (1) 证明：连接  $OD$ ，则  $OD=OE$ ，

$\therefore \angle ODE = \angle BEF, \dots\dots\dots 2 \text{分}$

$\because BF=BE,$

$\therefore \angle F = \angle BEF,$

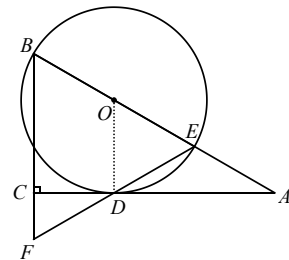
$\therefore \angle ODE = \angle F,$

$\therefore OD \parallel BF, \dots\dots\dots 4 \text{分}$

$\therefore \angle ADO = \angle ACB = 90^\circ ,$

$\because OD$  是  $\odot O$  的半径，且  $AC \perp OD$ ，

$\therefore AC$  是  $\odot O$  的切线.  $\dots\dots\dots 6 \text{分}$



(2) 解：  $\because \angle ADO = \angle ACB = 90^\circ , \angle A = 30^\circ , BC = 6,$

$\therefore \angle AOD = 90^\circ - \angle A = 60^\circ , AB = 2BC = 12,$

$\because OD = OE,$

$\therefore \triangle DOE$  是等边三角形, .....7 分

$$\therefore OB=OE=DE, \quad \angle OED=60^\circ,$$

$$\therefore \angle EDA = \angle OED - \angle A = 30^\circ = \angle A,$$

$$\therefore AE=DE,$$

$$\therefore OB=OE=AE=DE=\frac{1}{3}AB=4,$$

$$\therefore BE=2OE=8, \quad \dots\dots\dots 8 \text{ 分}$$

$\because BE$  是  $\odot O$  的直径,

$$\therefore \angle BDE=90^\circ,$$

$$\therefore BD = \sqrt{BE^2 - DE^2} = \sqrt{8^2 - 4^2} = 4\sqrt{3},$$

$$\therefore S_{\triangle BED} = \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3},$$

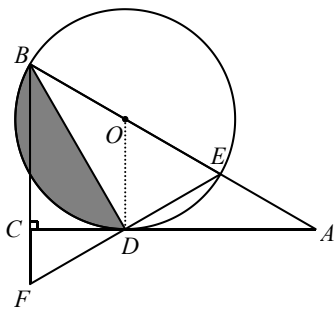
$$\therefore S_{\triangle BOD} = S_{\triangle EOD} = \frac{1}{2} S_{\triangle BED} = 4\sqrt{3}, \quad \dots\dots\dots 10 \text{ 分}$$

$$\therefore \angle OBD = 2\angle OED = 120^\circ,$$

$$\therefore S_{\text{扇形 } BOD} = \frac{120\pi \times 4^2}{360} = \frac{16}{3}\pi,$$

$$\therefore S_{\text{阴影}} = S_{\text{扇形 } BOD} - S_{\triangle BOD} = \frac{16}{3}\pi - 4\sqrt{3},$$

$\therefore$  阴影部分的面积为  $\frac{16}{3}\pi - 4\sqrt{3}$ . .....12 分



22. (1) 解: 把  $a = -1$  代入  $y = (a + 2)x^2 - 2(a - 1)x + a - 5 = x^2 + 4x - 6$

当  $x=0$  时,  $y=-6$

因此, 抛物线与  $y$  轴的交点坐标为  $(0, -6)$  .....3 分

(2) 解: 由题意得, 抛物线的对称轴是直线  $x = \frac{a-1}{a+2} = \frac{4-m+m}{2} = 2$ ,  $\therefore a = -5$

抛物线关系式为:  $y = -3x^2 + 12x - 10$ , ..... 5 分

$\therefore$  抛物线开口向下, 点  $C(4, q)$  关于对称轴的对称点是  $(0, q)$  .....6 分

$\because p > q \therefore 0 < 4 - m < m < 4$  或  $0 < m \leq 4 - m < 4$ ,

$\therefore 2 < m < 4$  或  $0 < m \leq 2$  即  $0 < m < 4$  .....8 分

(3) 解:  $y = -3x^2 + 12x - 10 = -3(x-2)^2 + 2$ , 对称轴为直线  $x = 2$ , .....9 分

① 当  $t-1 \leq 2t+3 \leq 2$  时,  $-4 \leq t \leq -\frac{1}{2}$ ,  $-3(2t+3-2)^2 + 2 = 1$ , 得

$$t_1 = -\frac{1}{2} - \frac{\sqrt{3}}{6}, t_2 = -\frac{1}{2} + \frac{\sqrt{3}}{6} \text{ (舍去)} \dots\dots\dots 10 \text{ 分}$$

② 当  $t-1 \leq 2 < 2t+3$  时,  $y_{\text{最大值}} = 2$ , 此种情况不成立 .....11 分

③ 当  $2 \leq t-1 \leq 2t+3$  时,  $t \geq 3$ ,  $-3(t-1-2)^2 + 2 = 1$ , 得

$$t_1 = 3 + \frac{\sqrt{3}}{3}, t_2 = 3 - \frac{\sqrt{3}}{3} \text{ (舍去)} \dots\dots\dots 12 \text{ 分}$$

综上所述,  $t = -\frac{1}{2} - \frac{\sqrt{3}}{6}$  或  $t = 3 + \frac{\sqrt{3}}{3}$  .....13 分

23.

(1) 解: 由题意得,  $\angle B = \angle C = \angle NEM, BE = CE = \frac{1}{2}BC = \frac{1}{2} \times 6 = 3$  .....1 分

$\therefore \angle NEC = \angle B + \angle BNE = \angle NEM + \angle CEM$

$\therefore \angle BNE = \angle CEM \therefore \triangle BNE \sim \triangle CEM$  .....3 分

$\therefore \frac{BN}{CE} = \frac{BE}{CM} \therefore BN \cdot CM = CE \cdot BE = 3 \times 3 = 9$  .....4 分

(2) ①解：由题意得  $\angle B = \angle C = \angle DEF = \angle F, \angle BAC = \angle D$  .....5 分

$\therefore AP \parallel DF \therefore \angle EAP = \angle D, \angle APE = \angle F = \angle DEF \therefore AE = AP$  .....6 分

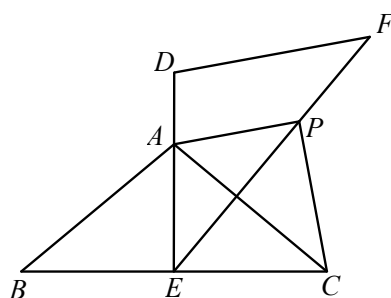
$\therefore \angle BAC = \angle EAP, \therefore \angle BAE = \angle CAP,$

又  $\therefore AB = AC, \therefore \triangle BAE \cong \triangle CAP \therefore AP = AE, CP = BE,$  .....7 分

当  $AE \perp BC$  时,  $AE$  长最小,  $AP$  长就最小 .....8 分

$\therefore AB = AC, AE \perp BC \therefore BE = \frac{1}{2} BC = 3$

因此,  $CP = BE = 3$  .....9 分



② 证明  $\therefore DF \parallel BC \therefore \angle F = \angle FEC, \therefore \angle F = \angle B \therefore \angle B = \angle FEC \therefore PE \parallel AB$  .....10 分

又  $\therefore AP \parallel DF \parallel BC \therefore$  四边形  $ABEP$  是平行四边形  $\therefore AB = EP = DF$

$\therefore AP \parallel DF \therefore \frac{AP}{DF} = \frac{EP}{EF} \therefore AP \cdot EF = DF \cdot EP$  .....12 分

$\therefore EP = DF, EF = BC \therefore DF^2 = AP \cdot BC$  .....13 分