

2026 年初中学业水平考试

数学模拟试题参考答案及评分标准

一、选择题(每题 4 分, 共 40 分)

BBCDC BABCD

二、填空题(每题 4 分, 共 20 分)

11. -1 12. 北偏西 50° , 5km 处 13. $\frac{9}{8}$ 14. $2\sqrt{3}$ 15. ①②③

三、解答题(本大题共 8 个小题, 共 90 分)

16. (满分 10 分)

解: (1) 原式 $= 4 + 2\sqrt{3} + \sqrt{3} - 1 - 6$

$$= 3\sqrt{3} - 3; \dots\dots\dots 5 \text{ 分}$$

$$\begin{aligned} (2) \text{ 原式} &= \frac{1}{(m+7)(m-7)} \cdot m(m-7) + \frac{3m}{m+7} \\ &= \frac{m}{m+7} + \frac{3m}{m+7} \dots\dots\dots 8 \text{ 分} \\ &= \frac{4m}{m+7} \dots\dots\dots 10 \text{ 分} \end{aligned}$$

17. (10 分)

解: (1) 由题可知 $\angle\alpha = 90^\circ - 63^\circ = 27^\circ$,
故答案为: 27° . $\dots\dots\dots 3 \text{ 分}$

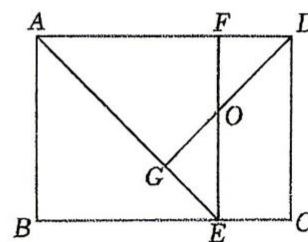
(2) 由题意可得 $AM = EN = 1.6m$, $MN = AE = 10m$, $\dots\dots\dots 5 \text{ 分}$
在直角三角形 ADE 中, $DE = AE \cdot \tan\alpha = 10 \times \tan 27^\circ \approx 10 \times 0.51 = 5.1m$, $\dots\dots\dots 8 \text{ 分}$
 $\therefore DN = DE + EN = 5.1 + 1.6 = 6.7m$,
 \therefore 结果保留整数, 即 $DN \approx 7m$, $\dots\dots\dots 10 \text{ 分}$

答: 大树的高度约为 $7m$.

18. (12 分)

(1) 证明: \because 矩形 $ABCD$,
 $\therefore \angle BAF = \angle ABE = 90^\circ$,
 $\because EF \perp AD$, \therefore 四边形 $ABEF$ 是矩形,
 $\because AE$ 平分 $\angle BAD$, $\therefore EF = EB$,
 \therefore 四边形 $ABEF$ 是正方形; $\dots\dots\dots 4 \text{ 分}$

(2) 证明: $\because AE$ 平分 $\angle BAD$,
 $\therefore \angle DAG = \angle BAE$,



$$\text{在}\triangle AGD\text{和}\triangle ABE\text{中,}\begin{cases} \angle DAG = \angle BAE \\ \angle AGD = \angle ABE, \\ AD = AE \end{cases}$$

$\therefore \triangle AGD \cong \triangle ABE$ (AAS),

$\therefore AB = AG$;8分

(3) 解: \because 四边形 $ABCD$ 是矩形, $\therefore \angle BAF = \angle ABE = 90^\circ$,

$\because EF \perp AD$, \therefore 四边形 $ABEF$ 是矩形,

$\because AE$ 平分 $\angle BAD$, $\therefore EF = EB$, $\angle BAE = \angle DAG = 45^\circ$,

\therefore 四边形 $ABEF$ 是正方形;10分

$\therefore AB = AF = 1$,

$\because \triangle AGD \cong \triangle ABE$, $\therefore DG = AB = AF = AG = 1$,

$\therefore AD = \sqrt{2}$, $\angle DAG = \angle ADG = 45^\circ$, $\therefore DF = \sqrt{2} - 1$,

$\because EF \perp AD$, $\therefore \angle FDO = \angle FOD = 45^\circ$, $\therefore DF = OF = \sqrt{2} - 1$.

$\therefore OF = \sqrt{2} - 1$12分

19. (10分)

解: (1) $360^\circ \times 20\% = 72^\circ$,

$\therefore D$ 组对应的扇形的圆心角为 72° ;3分

(2) 具体为: 18.1, 18.3, 18.5, 18.5, 18.8, 18.8, 18.8, 18.9. 众数为: 18.8,

故答案为: 18.8,5分

不正确.

理由: 仅抽取的 8 个样本的众数不等于 D 组的 200 个数据的众数, 不能判断众数, 故不正确;7分

(3) 乙品种芒果的品质更优.

理由: 甲品种芒果的优级产品所占的百分比为:

$20\% + 25\% = 45\%$;8分

乙品种芒果的优级产品所占的百分比为:

$$\frac{50 + 70}{200} \times 100\% = 60\%.$$

$\because 60\% > 45\%$,10分

\therefore 乙品种芒果的品质更优.

20. (10分)

解: (1) 设 B 型机器人每小时搬运 x kg 材料,

根据题意列分式方程得, $\frac{1000}{x+30} = \frac{600}{x}$,3分

解得 $x=45$,5分

经检验, $x=45$ 是所列方程的解且符合题意:6分

当 $x=45$ 时, $x+30=75$,

答: A 型机器人每小时搬运 75kg 材料, B 型机器人每小时搬运 45kg 材料;

(2) 设购进 A 型机器人 a 台, 则购进 B 型机器人 $(20 - a)$ 台,

则有 $75a+45(20 - a) \geq 1400$,8分

解得 $a \geq \frac{50}{3}$,9分

$\because a$ 是整数,

$\therefore a \geq 17$;10分

答: 至少购进 A 型机器人 17 台.

21. (10分)

证明: (1) 过点 O 作 $OH \perp CD$, H 为垂足,

\because 在平行四边形 $ABCD$ 中, $AE \perp BC$, $AD \parallel BC$,

$\therefore \angle BEA = \angle EAD = 90^\circ$, $\therefore OA \perp AD$,

$\because DO$ 平分 $\angle ADC$, $\therefore OH = OA$,

$\therefore OH$ 是半径,

$\because OH \perp CD$,

\therefore 直线 CD 是 $\odot O$ 的切线;5分

(2) 连接 OB , 设 $\odot O$ 的半径是 r ,

$$\because \tan \angle ABC = \frac{AE}{BE} = \frac{3}{2}, BE=4,$$

$$\therefore AE=6, \therefore OE=6-r,$$

在 $Rt\triangle OBE$ 中, 根据勾股定理得 $r^2 = (6-r)^2 + 16$,

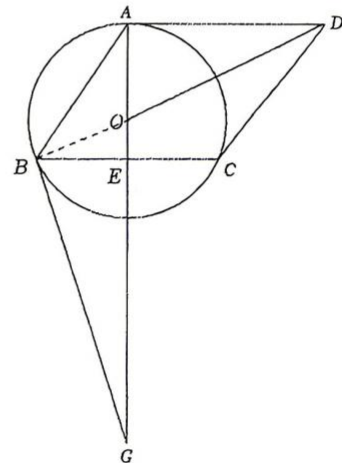
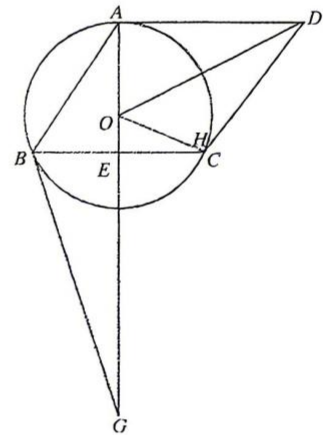
$$\text{解得 } r = \frac{13}{3}, \text{7分}$$

$$\therefore OE = \frac{5}{3}, OB = \frac{13}{3},$$

$\because BG \perp OB$,

$\therefore \angle OBG = 90^\circ$,

$\therefore \angle OBG = \angle OEB = 90^\circ$, 又 $\angle BOG = \angle EOB$,



$$\therefore \triangle OBG \sim \triangle OEB, \therefore \frac{BG}{EB} = \frac{OB}{OE}, \therefore \frac{BG}{4} = \frac{\frac{13}{5}}{\frac{3}{3}},$$

$$\therefore BG = \frac{52}{5}. \dots\dots\dots 10 \text{分}$$

22. (14分)

解: (1) $\because y = x^2 - 2ax + 3 - a^2 = (x - a)^2 + 3 - 2a^2,$

\therefore 顶点坐标 $(a, 3 - 2a^2), \dots\dots\dots 2 \text{分}$

\because 抛物线的顶点恰好在直线 $y = 2x - 1$ 上,

$\therefore 3 - 2a^2 = 2a - 1,$ 解得: $a_1 = 1, a_2 = -2. \dots\dots\dots 4 \text{分}$

(2)

方法 1: 当 $a > 0$ 时, $y = x^2 - 2x + 2,$ 点 (x_0, y_0)

是抛物线上一点, $\therefore y_0 = x_0^2 - 2x_0 + 2,$

\therefore 由 $\begin{cases} y = 2x - 1 \\ y = x^2 - 2x + 2 \end{cases},$ 解得: $\begin{cases} x_1 = 1 \\ y_1 = 1 \end{cases}, \begin{cases} x_2 = 3 \\ y_2 = 5 \end{cases} \dots\dots\dots 6 \text{分}$

\therefore 可设直线与抛物线的交点为 $A(1, 1)$ 和 $B(3, 5),$

当 $1 < x_0 < 3$ 时, $P(x_0, y_0)$ 在线段 AB 的下方, $\therefore y_0 < 2x_0 - 1$

$\therefore |2x_0 - y_0 - 1| = 2x_0 - y_0 - 1 = -x_0^2 + 4x_0 - 3 = -(x_0 - 2)^2 + 1 \leq 1 \dots\dots\dots 9 \text{分}$

方法 2: 当 $a > 0$ 时, $y = x^2 - 2x + 2,$ 点 (x_0, y_0) 是抛物线上一点,

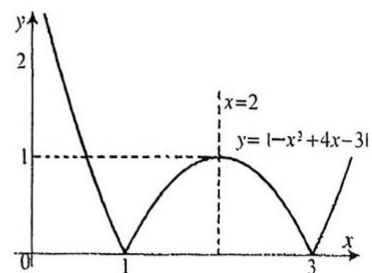
$\therefore y_0 = x_0^2 - 2x_0 + 2, \therefore |2x_0 - y_0 - 1| = |-x_0^2 + 4x_0 - 3| = |-(x_0 - 2)^2 + 1|$

不妨令 $y = |-(x_0 - 2)^2 + 1|,$

它的图像如图, 当 $1 < x_0 < 3$ 时,

y 的最大值=1,

$\therefore |2x_0 - y_0 - 1| \leq 1 \dots\dots\dots 9 \text{分}$



(3) $\because y = x^2 - 2ax + 3 - a^2 = (x - a)^2 + 3 - 2a^2$,

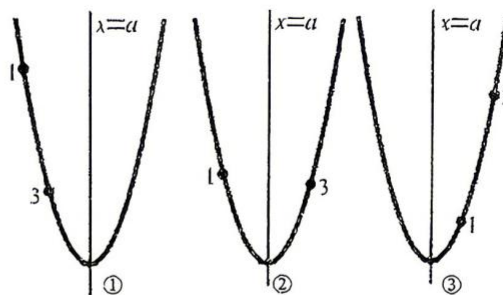
第①种情况：当 $a \geq 3$ 时，当 $x = 3$ 时，

$y_{\text{有最小值}} = 4$,

$\therefore 9 - 6a + 3 - a^2 = 4$

解得： $a_1 = -3 - \sqrt{17}$ ， $a_2 = -3 + \sqrt{17}$ ，

$\because a > 3$ ， $\therefore a = -3 \pm \sqrt{17}$ 不符合题意舍去；



第②种情况：当 $1 < a < 3$ 时，函数在顶点处取得最小值 4，

$\therefore 3 - 2a^2 = 4$ ，

即 $2a^2 + 1 = 0$ 该方程无实根；

第③种情况：当 $a \leq 1$ 时，当 $x = 1$ 时， $y_{\text{有最小值}} = 4$ ，

$\therefore 1 - 2a + 3 - a^2 = 4$

解得： $a_1 = -2$ ， $a_2 = 0$ ，

综合①②③， $\therefore a_1 = -2$ ， $a_2 = 0$ 14分

23. (14分)

解：(1) 设 $\angle EAB = \alpha$ ，

\because 正方形 $ABCD$ ， $\therefore AB = AD$ ， $\angle BAD = 90^\circ$ ，

$\therefore \angle AED = \angle ADE = \frac{180^\circ - (90^\circ + \alpha)}{2} = 45^\circ - \frac{1}{2}\alpha$ ，

\because 将边 AB 绕点 A 旋转到 AE ， $\therefore AE = AB$ ，

$\therefore \angle AEB = \angle ABE = \frac{180^\circ - \alpha}{2} = 90^\circ - \frac{1}{2}\alpha$ ，

$\therefore \angle DEB = (90^\circ - \frac{1}{2}\alpha) - (45^\circ - \frac{1}{2}\alpha) = 45^\circ$ 。

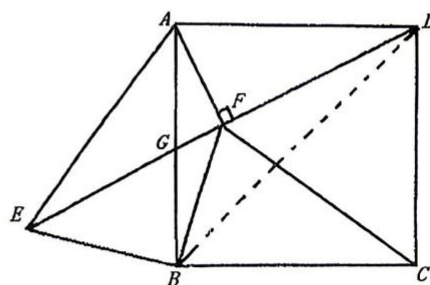
.....4分

(2) 连接 BD ，

\because 正方形 $ABCD$ ，

$\therefore \angle ABF + \angle FBD = 45^\circ$ ， $\angle BCD = 90^\circ$ ，

$\because BC = CF = CD$ ，



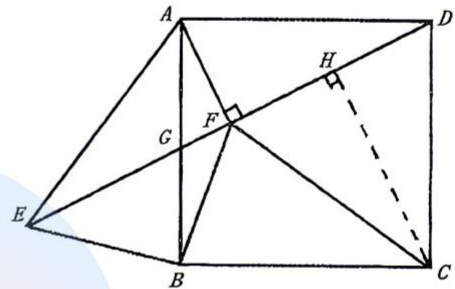
$$\begin{aligned} \therefore \angle BFD &= \angle BFC + \angle CFD \\ &= \frac{180^\circ - \angle BCF}{2} + \frac{180^\circ - \angle DCF}{2} = 180^\circ - \frac{\angle BCF + \angle FCD}{2} \\ &= \frac{180^\circ - \angle BCD}{2} = 180^\circ - 45^\circ = 135^\circ, \dots\dots\dots 6 \text{分} \end{aligned}$$

$\therefore \angle FBD + \angle FDB = 45^\circ$, $\angle EFB = 45^\circ$, $\therefore \angle ABF = \angle FDB$,
又 $\because AF \perp DE$, $\therefore \angle AFB = 90^\circ + 45^\circ = 135^\circ$, $\therefore \angle AFB = \angle BFD$, $\therefore \triangle BAF \sim \triangle DBF$,

$$\therefore \frac{BF}{AF} = \frac{DF}{BF}, \text{ 即 } BF^2 = AF \cdot FD. \dots\dots\dots 10 \text{分}$$

(3) 过点 C 作 $CH \perp DE$, 垂足为 H,

$\therefore \angle HDC + \angle DCH = 90^\circ$, $\angle AFD = \angle CHD = 90^\circ$,
又 $\because \angle ADF + \angle HDC = 90^\circ$, $\therefore \angle ADF = \angle DCH$,
又 $\because AD = CD$, $\therefore \triangle AFD \cong \triangle DHC$,
 $\therefore AF = DH$, $FD = CH$,
又 $\because CF = CD$, $\therefore FH = HD$,
 \therefore 令 $AF = k$, $\therefore FD = 2k$,



由(2)可得: $BF^2 = AF \cdot FD = 2k^2$,
 $\therefore \angle FEB = 45^\circ$, $\angle EFB = 45^\circ$,

$$\therefore \angle EBF = 90^\circ, \text{ 且 } EB = FB, \therefore S_{\triangle BEF} = \frac{1}{2} BE \cdot BF = \frac{1}{2} BF^2 = k^2$$

$$\because AF = k, \therefore FD = 2k, \therefore AD = \sqrt{AF^2 + FD^2} = \sqrt{k^2 + (2k)^2} = \sqrt{5}k,$$

$$\begin{aligned} \because \angle GAD = \angle AFD = 90^\circ, \angle ADF = \angle ADF \\ \therefore \triangle DAG \sim \triangle DFA, \therefore \frac{AG}{AD} = \frac{FA}{FD} = \frac{1}{2}, \therefore AG = \frac{\sqrt{5}}{2}k, \end{aligned}$$

$$\begin{aligned} \therefore S_{\text{四边形}CFGB} &= S_{\text{正方形}ABCD} - S_{\triangle AGD} - S_{\triangle CFD} \\ &= (\sqrt{5}k)^2 - \frac{1}{2} \cdot \frac{\sqrt{5}}{2}k \cdot \sqrt{5}k - \frac{1}{2} \cdot 2k \cdot 2k = \frac{7}{4}k^2 \end{aligned}$$

$$\therefore \frac{S_{\triangle BEF}}{S_{\text{四边形}CFGB}} = \frac{k^2}{\frac{7}{4}k^2} = \frac{4}{7} \dots\dots\dots 14 \text{分}$$